49TH ANNUAL AUGUSTA UNIVERSITY MATHEMATICS CONTEST

INDIVIDUAL EXAM

Time: 1 hour | MARCH 1, 2024 | Maximum Points: 20

WRITTEN PART INSTRUCTIONS
(to be read aloud to students by proctor)

Use a #2 pencil only. Do not use a pen or a colored pencil. Turn your answer sheet so that the word "Name" is at the top. Print your last name, first name, and middle initial in the space provided and today's date, which is March 1, 2024. Print the name of your school in the space marked "School", and indicate whether you are on Team A, Team B, or an Alternate in the space marked "Test Name".

This is a multiple-choice test. You are to mark only one answer to each question. Answers must be recorded on the separate answer sheet. Select the one best answer to each question and record this answer by blackening the appropriate space of the letter on the answer sheet. For example, if you select Option B for the first question, then you would blacken the "B" on the answer sheet for that first question. There is no penalty for guessing. Do not make any stray marks or calculations on the answer sheet.

You may do any scratch work in the space provided on these pages. Do as many problems as you can in the 60 minutes allowed. When the alarm sounds on the timer or when the proctor requests you to stop, please cease work immediately, put your pencil down, and turn your answer sheet over.

Essentially all of the problems require some work to solve them. Do not be hasty in your judgments, but work as rapidly as possible. For each problem you should work out ideas on paper before selecting the answer. Do not assume that any of the drawings are necessarily to scale.

There are 20 questions on the test. Check your test carefully before starting. If a question or page is missing, raise your hand and the proctor will provide you with a correct copy of the test.

In case of ties, the answers to Number 20 will be examined to break the tie. If there is still a tie, the answers to Number 19 will be examined, etc.

The proctor is not permitted to explain to you the meaning of any question, so do not request your proctor to break the rules of the contest. The use of calculators is prohibited. If you have questions concerning the instructions, ask them now.

DO NOT TURN THE PAGE UNTIL THE PROCTOR TELLS YOU TO BEGIN WORK.

Please go on to the next page...

1) A deck of fifty-two cards is divided into 4 suits (clubs, diamonds, hearts, spades), and each suit contains 10 cards numbered 1 through 10 (Ace = 1) and 3 face cards (jack, queen, king). If a card is drawn at random from the deck, what is the probability of drawing a diamond or a card with a number divisible by three?

a) $\frac{1}{2}$

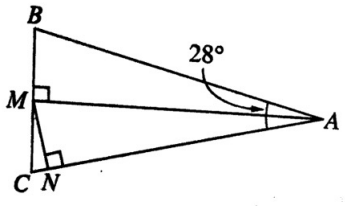
b) $\frac{11}{26}$

c) $\frac{17}{26}$

d) $\frac{25}{52}$

e) $\frac{33}{52}$

2) If $\triangle$ABC is isosceles, $\angle CAB = 28°$, $AM \perp BC$ and $MN \perp AC$, what is the measure of $\angle CMN$?



a) 30°

b) 18°

c) 15°

d) 26°

e) 14°

3) If $\log_7 x = \frac{1}{2}\log_7 5 + 3$, find x.

a) $5^7\sqrt{3}$

b) $5^3\sqrt{7}$

c) $3^7\sqrt{5}$

d) $7^3\sqrt{5}$

e) $7^5\sqrt{3}$

4) What is the greatest common divisor of 1120 and 189?

a) 5
b) 7
c) 9
d) 3
e) None of the above

5) If $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$, what is $\tan\left(\frac{\pi}{8}\right)$?

a) $\frac{\sqrt{2+\sqrt{2}}}{2}$
b) $\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$
c) $\frac{2}{\sqrt{2-\sqrt{2}}}$
d) $\frac{\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}$
e) None of the above

6) A lecture hall has six seats in the first row, eight in the second, ten in the third, and so on through row 12. Rows 12 through 20 (the last row) all have the same number of seats. What is the number of seats in the lecture hall?

a) 258
b) 224
c) 482
d) 428
e) None of the above

7) In a sandwich shop, a customer may choose to have or omit any of the following on a sandwich: mustard, ketchup, onion, pickle, tomato, relish, mayonnaise, lettuce. How many different choices are there for a sandwich order?

a) 66
b) 130
c) 256
d) 288
e) None of the above

8) In a game two players take turns tossing a fair coin. The winner is the first one to toss a head. What is the probability that the player who makes the first toss in the game becomes the winner?

a) $\frac{1}{3}$

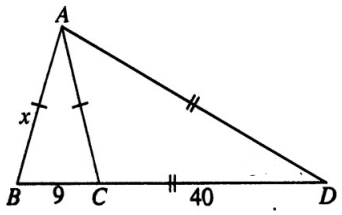
b) $\frac{2}{3}$

c) $\frac{1}{2}$

d) $\frac{1}{4}$

e) $\frac{3}{4}$

9) If $AB = AC$, $AD = BD$, $BC = 9$ and $CD = 40$, what is the value of x?



a) 7

b) 21

c) 10

d) 24.5

e) 15

10) A given high school has 1500 students. Of those students, 800 like football. 200 of those students who like football also like basketball. The group of students who like both football and basketball makes up one third of the total number of students who like basketball. How many students like a sport other than football and basketball?

a) 250

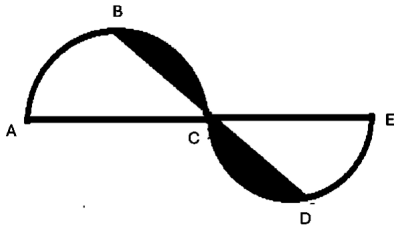
b) 300

c) 350

d) 400

e) None of the above

11) In the given figure, the semicircles are identical and have radius of 1, B is the midpoint of the arc ABC, and D is the midpoint of the arc CDE. What is the area of the shaded region?



a) $\pi - 1$

b) $\dfrac{3\pi}{2} - 1$

c) $\dfrac{\pi}{2} - 1$

d) $\pi - 2$

e) None of the above

12) What does $\left(\dfrac{\cos^2 x + 1}{3\sin x}\right)^2 \left(\dfrac{\sin^2 x}{4(\sin^2 x + \cos^2 x)}\right)^2 \left(\dfrac{3\sin x}{1 - \cos^4 x}\right)^2$ equal to?

a) $\dfrac{1}{16}$

b) $\dfrac{1}{4}$

c) $\dfrac{3}{4}$

d) $\dfrac{3}{16}$

e) None of the above

13) What is the value of $\sin\left(\dfrac{5\pi}{12}\right)$?

a) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

b) $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

c) $\dfrac{\sqrt{6} - \sqrt{2}}{2}$

d) $\sqrt{6} - \sqrt{2}$

e) $\sqrt{6} + \sqrt{2}$

14) A student is required to solve 6 out of 10 questions on an exam. The questions are divided into two sections of 5 questions each. In how many ways can the student select the questions to solve if no more than 4 questions can be selected from either section?

a) 100
b) 110
c) 150
d) 200
e) 210

15) The side length of a cube is 5 cm. If all of the sides of the cube are painted, and it is sliced into smaller cubes with volume 1 cm^3, how many of those smaller cubes will have exactly one of their sides painted?

a) 9
b) 54
c) 61
d) 64
e) None of the above

16) If the surface area of a cube is twice as large as its volume, find the cube's volume.

a) 9
b) 27
c) 36
d) 54
e) 81

17) A movie theater sold tickets for a movie at \$7 for a showing earlier in the day and \$11 for a showing later in the day, and the number of \$7 tickets sold was different than the number of \$11 tickets sold. A fourth of all those who purchased a ticket also spent \$4 each on popcorn at the movie theater. If the total amount from tickets and popcorn sold was \$124, how many \$7 tickets were sold?

a) 2
b) 5
c) 8
d) 12
e) 16

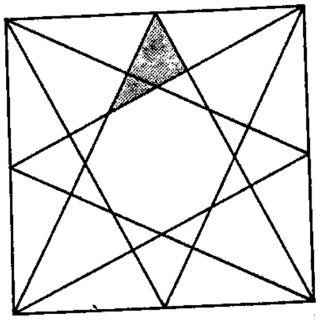
18) The product of the roots of the equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\left(\frac{5}{4}\right)} = \sqrt{2}$ is $1/\sqrt[b]{a}$, where b and a are positive integers. What is the value of $(a+b)$?

a) 9
b) 11
c) 17
d) 19
e) None of the above

19) The financial department of a local clothing store reported that 30% of purchases are paid by cash, 30% are paid by check, and 40% are paid by a credit card. The department also reported that 20% of the purchases paid by cash, 60% of the purchases paid by check, and 80% of the purchases paid by credit card are for more than \$50. William just purchased a new jacket that cost \$60. What is the probability that William paid with cash?

a) $\frac{3}{50}$
b) $\frac{3}{28}$
c) $\frac{1}{5}$
d) $\frac{3}{25}$
e) $\frac{3}{10}$

20) How many of the triangles in the figure below are similar to the shaded triangle?



a) 7
b) 12
c) 20
d) 24
e) 30