1. (8 pts.) Find and simplify the difference quotient \( \frac{f(x+h) - f(x)}{h}, h \neq 0 \) for the function \( f(x) = 3x + 2 \).

\[
\begin{align*}
f(x+h) &= 3(x+h) + 2 = 3x + 3h + 2 \\
f(x+h) - f(x) &= (3x + 3h + 2) - (3x + 2) = 3h \\
\frac{f(x+h) - f(x)}{h} &= \frac{3h}{h} = \boxed{3}
\end{align*}
\]

2. (10 pts.) The graph of the function \( f(x) \) is shown below. On the same xy-plane, use transformation rules to graph \( y = 2f(x) + 2 \). Label three points on the graph of the transformed function.
3. (10 pts.) Find the inverse function of \( f(x) = \frac{4x - 1}{x} \).

Switch and solve

\[
x = \frac{4y - 1}{y}
\]

\[
x = \frac{4y - 1}{y} = \frac{4y}{y} - \frac{1}{y} = 4 - \frac{1}{y}
\]

\[
x = \frac{4y - 1}{y} = 4 - \frac{1}{y}
\]

\[
x = \frac{4y - 1}{y} = 4 - \frac{1}{y}
\]

\[
\frac{1}{y} = \frac{4y - 1}{y} - 4 = \frac{4y - 1 - 4y}{y} = \frac{-1}{y}
\]

\[
f^{-1}(x) = \frac{-1}{x - 4} \quad \text{or} \quad \frac{1}{4 - x}
\]

4. Given \( P(x) = -(x - 4)^2 (x^2 - 25) = -(x - 4)^2 (x - 5)(x + 5) \)

a) (2 pt.) What is the degree of the function? 4

b) (9 pts.) Determine the zeros of the function. State the multiplicity of each zero and if the graph crosses the x-axis or touches the x-axis and turns around, at each zero.

<table>
<thead>
<tr>
<th>Zero</th>
<th>Multiplicity</th>
<th>Behavior of the graph at the zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>a. Touches the x-axis and turns around</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Crosses the x-axis</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>a. Touches the x-axis and turns around</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Crosses the x-axis</td>
</tr>
<tr>
<td>-5</td>
<td>1</td>
<td>a. Touches the x-axis and turns around</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Crosses the x-axis</td>
</tr>
</tbody>
</table>

5. (10 pts.) An artifact originally had 16 grams of carbon-14 present. The decay model, \( A(t) = 16 e^{-0.000121t} \) describes the amount of carbon-14 present after \( t \) years. How long will it take for the amount of carbon-14 in the artifact to decay to 8 grams? Round the answer to the nearest whole number.

\[
8 = 16 e^{-0.000121t}
\]

\[
\frac{8}{16} = e^{-0.000121t}
\]

\[
ln\left(\frac{8}{16}\right) = -0.000121t
\]

\[
t = \frac{\ln\left(\frac{8}{16}\right)}{-0.000121} \approx 5728 \text{ years}
\]
6. Given the rational function \( r(x) = \frac{2x + 8}{(x + 3)(x - 4)} \)

a. (2 pts.) Find the vertical asymptotes of the graph of \( r(x) \), if any.
   \[ x = -3, 4 \]

b. (2 pts.) Find the x intercepts of the graph of \( r(x) \).
   \[ x = -4 \]

c. (2 pt.) Find the y intercept of the graph of \( r(x) \).
   \[ y = -\frac{2}{3} \]

d. (1 pt.) Find the horizontal asymptote of the graph of \( r(x) \), if there is one.
   \[ y = 0 \]

e. (5 pts.) Use parts (a – d) to sketch the graph of \( r(x) \). Label the vertical and horizontal asymptotes.

7. (10 pts.) Use the properties of logarithms to expand the logarithmic expression as much as possible.

\[ \log \left( \frac{(x+1)^3 \sqrt{y}}{x^2 y} \right) \]

\[ 3 \log(x+1) + \frac{1}{2} \log y - 2 \log x \]
8. Find the exact value without using a calculator.
   
   a. (5 pts.) \( e^{\ln(\pi)} = \pi \)

   b. (5 pts.) \( \log_{10} \sqrt{3} = \sqrt{3} \)

9. (10 pts.) Solve the logarithmic equation \( \log_{2}(x+30) - \log_{2}(2x) = 3 \).

   \[
   \log_{2} \left( \frac{x+30}{2x} \right) = 3
   \]

   \[
   2^{3} = \frac{x+30}{2x}
   \]

   \[
   8 = \frac{x+30}{2x}
   \]

   \[
   8x = x + 30
   \]

   \[
   16x = 30
   \]

   \[
   x = \frac{30}{16} = \frac{15}{8}
   \]

10. Answer the following:
   
   a. (5 pts.) Draw the angle \( \theta = \frac{17\pi}{6} \) in standard position.

   b. (5 pts.) State an angle between 0 and 2\( \pi \) which is coterminal to the angle \( \theta \).

   \[
   \frac{5\pi}{6}
   \]

11. (10 pts.) From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot. Round to the nearest whole number.

   \[
   \tan 57.2^\circ = \frac{x}{125}
   \]

   \[
   x = 125 \tan 57.2^\circ \approx 194
   \]

   \[
   \theta = 57.2^\circ
   \]

   125 ft.
12. (7 pts.) Find the measure in radians of a central angle \( \theta \) subtended by an arc of length 7 centimeters in a circle of radius 3 centimeters. State your answer to the nearest tenth of a radian.

\[
S = (\theta \Rightarrow \theta = \frac{\theta}{\pi} = \frac{7}{3} \approx 2.3 \text{ radians}
\]

13. Find the exact value of the following. Show work.

a. (5 pts.) \( \cos \left( \frac{5\pi}{3} \right) = 2 \)

b. (5 pts.) \( \sin \left( \frac{7\pi}{6} \right) \)

14. Find the exact values of the following expressions. Show work.

a. (5 pt.) \( \sin^{-1} \left( \sin \left( \frac{5\pi}{6} \right) \right) = \frac{\pi}{6} \)

b. (5 pt.) \( \cos \left( \tan^{-1} \left( -\frac{2}{3} \right) \right) = \frac{3}{\sqrt{13}} \)
15. If \( \tan(t) = \frac{-8}{15} \), and \( 270^\circ < t < 360^\circ \). Find the exact values of the following:

a. (4 pts.) \( \cos(t) = \frac{15}{17} \)

b. (4 pts.) \( \sin\left(\frac{t}{2}\right) \)

\[
270^\circ < t < 360^\circ \Rightarrow 135^\circ < \frac{t}{2} < 180^\circ \quad \frac{t}{2} \text{ in QII, so sine pos}
\]

\[
\sin\left(\frac{t}{2}\right) = \sqrt{\frac{1 - \cos t}{2}} = \sqrt{\frac{1 - \frac{15}{17}}{2}} = \sqrt{\frac{1}{17}} \text{ or } \frac{\sqrt{17}}{17}
\]

c. (4 pts.) \( \cos(2t) \)

\[
\cos^2 t - \sin^2 t = \left(\frac{15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2
\]

\[
\frac{225}{289} - \frac{64}{289} = \frac{161}{289}
\]

16. (10 pts.) Use an addition identity to show that \( \sin(\theta + \pi) = -\sin \theta \).

\[
\sin(\theta + \pi) = \sin \theta \cos \pi + \cos \theta \sin \pi
\]

\[
\cos \pi = -1 \quad \sin \pi = 0
\]

\[
\sin \theta (-1) + \cos \theta (0)
\]

\[-\sin \theta\]
17. Given the trigonometric function $f(x) = 2 \cos (2x - 3\pi)$

a. (2 pt.) Determine the amplitude of $f(x)$. 
   
   \[
   \begin{array}{c|c}
   x & \cos \omega x \\
   \hline
   3\pi/2 & 2 \\
   2\pi + \pi/4 & 0 \\
   2\pi & -2 \\
   9\pi/4 & 0 \\
   5\pi/2 & 2 \\
   \end{array}
   \]

b. (2 pt.) Determine the period of $f(x)$. 
   
   \[
   \pi/2
   \]

c. (2 pt.) Determine the phase shift of $f(x)$. 
   
   \[
   \frac{3\pi}{2}
   \]

   \[
   2
   \]

d. (4 pt.) Sketch one period of the graph of $f(x)$. Label the x and y intercepts.

18. (10 pts.) Verify the following trigonometric identity $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$.

   \[
   \tan^2 x - \sin^2 x
   \]

   \[
   \frac{\sin^2 x}{\cos^2 x}
   \]

   \[
   \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}
   \]

   \[
   \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}
   \]

   \[
   \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}
   \]

   \[
   \frac{\tan^2 x \sin^2 x}{\cos^2 x}
   \]

   \[
   \frac{\sin^2 x \sin^2 x}{\cos^2 x}
   \]

   \[
   \tan^2 x \cdot \sin^2 x = \tan^2 x \cdot \sin^2 x
   \]

   \[
   \tan^2 x \cdot \sin^2 x
   \]
19. (10 pts.) Find all the *exact* solutions to the following equation on the interval \([0, 2\pi)\).

\[2\cos^2 x - \cos x - 1 = 0\]

\[(2\cos x + 1)(\cos x - 1) = 0\]

\[
\cos x = -\frac{1}{2} \quad \cos x = 1
\]

\[x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad x = 0\]

20. Alan, Bill and Casey are camping in their tents. If the straight line distance between Alan and Bill is 153 ft, the straight line distance between Bill and Casey is 175 ft, and the straight line distance between Alan and Casey is 201 ft.

a. (5 pts.) Use the Law of Cosines to find the angle between the straight lines joining Alan and Bill and Bill and Casey. State your answer to the nearest tenth of a degree.

\[b^2 = a^2 + c^2 - 2ac \cos B\]

\[\frac{b^2 - a^2 - c^2}{-2ac} = \cos B\]

\[B = \cos^{-1} \left( \frac{201^2 - 175^2 - 153^2}{-2(175)(153)} \right)\]

\[\approx 75.3^\circ\]

b. (5 pts.) Use the Law of Sines to find the angle between the straight lines joining Alan and Bill and Alan and Casey. State your answer to the nearest tenth of a degree.

Find angle \(A\)

\[\frac{\sin A}{175} = \frac{\sin 75.3^\circ}{201}\]

\[A \approx \sin^{-1} \left( \frac{175 \sin 75.3^\circ}{201} \right) \approx 57.3^\circ\]

(57.4° would also get full credit)