

FORTY-EIGHTH ANNUAL
AUGUSTA UNIVERSITY MATHEMATICS CONTEST
March 3, 2023

WRITTEN PART
INSTRUCTIONS

(to be read aloud to class by proctor)

Use a #2 pencil only; do not use a pen or a colored pencil. Turn your answer sheet so that the word "NAME" is at the top. Print your last name, first name, and middle initial in the space provided and today's date, which is March 3, 2023. Print the name of your school in the space marked "SUBJECT", and indicate whether you are on Team A, Team B, or an alternate.

Turn your answer sheet so that the numbers are in the proper order.

This is a multiple-choice test. You are to mark only one answer to each question. Answers must be recorded on the separate answer sheet. Select the one best answer to each question and record this answer by blackening the appropriate space of the letter on the answer sheet. For example, if you select Option B for the first question, then you would blacken the "B" on the answer sheet for that first question. There is no penalty for guessing. Do not make any stray marks or calculations on the answer sheet.

You may calculate in the space provided on these pages.

Do as many problems as you can in the 60 minutes allowed. When the alarm sounds on the timer or when the proctor requests you to stop, please cease work immediately, put your pencil down, and turn your answer sheet over.

Essentially all of the problems require some figuring. Do not be hasty in your judgments, but work as rapidly as possible. For each problem you should work out ideas on paper before selecting the answer. Do not assume that any of the drawings are necessarily to scale.

There are 20 questions on the test. Check your test carefully before starting. If a question or page is missing, raise your hand and the proctor will provide you with a correct copy of the test.

In case of ties, the answers to Number 20 will be examined to break the tie. If there is still a tie, the answers to Number 19 will be examined, etc.

The proctor is not permitted to explain to you the meaning of any question, so do not request your proctor to break the rules of the contest. The use of calculators is prohibited. If you have questions concerning the instructions, ask them now.

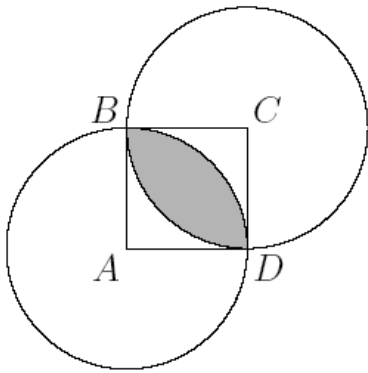
DO NOT TURN THE PAGE UNTIL THE PROCTOR TELLS YOU TO BEGIN WORK.

1. A picture was made on a copy machine. The copier had been set for a reduction to 80% of the original size. What percent will need to be used on the copier to obtain a full-size copy from the reduced sized picture?
 - a. 20%
 - b. 80%
 - c. 120%
 - d. 125%
 - e. None of the above.
2. Find the sum of all of the integers from -50 to 100 .
 - a. 1550
 - b. 2235
 - c. 3775
 - d. 4550
 - e. None of the above.
3. If the values of a, b, c , and d are 1, 2, 3, and 4, but not necessarily in that order, what is the largest possible value of the expression $ab + bc + cd + da$?
 - a. 17
 - b. 20
 - c. 23
 - d. 25
 - e. None of the above.
4. Bill and Linda each took some money from a piggy bank to buy an ice cream cone, but Bill was 24¢ short and Linda was 2¢ short of the price of a cone. They decided to pool their resources but found they still could not afford to buy the cone. How many cents did the ice cream cone cost?
 - a. 15
 - b. 18
 - c. 19
 - d. 20
 - e. 25
5. An area code is a positive 3-digit number. Add 7 to it and the result is divisible by 7. Add 8 instead, and the result is divisible by 8. Add 9 instead, and the result is divisible by 9. What is the area code?
 - a. 214
 - b. 504
 - c. 803
 - d. 919
 - e. None of the above

6. How many times must a pair of fair dice be thrown in order to be sure that there is some score that occurs at least three times?

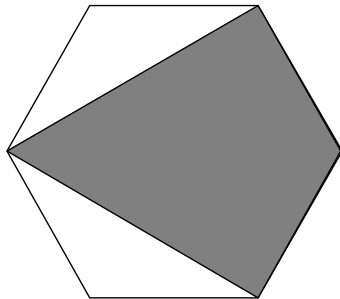
- a. 3
- b. 19
- c. 23
- d. 33
- e. 34

7. The square ABCD in the Figure has sides of length 1. Both of the circles pass through the points B and D, but one is centered at A and the other at C. What is the area of the intersection of the two circles (the shaded region in the Figure)?



- a. $\frac{\pi}{4}$
- b. $1 - \frac{\pi}{4}$
- c. $\frac{\pi}{2} - 1$
- d. $\frac{3\pi}{4} - 1$
- e. $\frac{1}{3}$

8. A company has 6,000 employees. Almost every one of the employees reads a local newspaper on a regular basis. The company town has three local newspapers, The Post, The Courier, and The Daily News. Seventy percent of the employees read The Post, forty-five percent read The Courier, and sixty percent read The Daily News. One fourth of the employees read The Post and The Courier, thirty percent read The Daily News and The Courier, and thirty five percent read The Post and The Daily News. One tenth of the people read all three papers on a regular basis. How many of these employees read none of the three local papers on a regular basis?
- a. 0
 - b. 150
 - c. 300
 - d. 450
 - e. None of the above.
9. Part of the following regular hexagon of side length 1 is shaded dark gray. What is the area of the *unshaded* portion of the hexagon? All angles and all sides of the hexagon are congruent.



- a. $\frac{\sqrt{3}}{2}$
- b. 1
- c. $\frac{\sqrt{3}+1}{4}$
- d. $\sqrt{3}$
- e. None of the above.

10. If $\sqrt{x} + \sqrt{y} = 17$ and $x - y = 85$, where x and y are real numbers, what is the value of x ?
- a. 25
 - b. 49
 - c. 121
 - d. 144
 - e. None of the above.
11. If $x = 1 + 3^a$ and $y = 1 + 3^{-a}$, which of the following is the correct expression for y in terms of x ?
- a. $y = x$
 - b. $y = x - 1$
 - c. $y = \frac{1}{x-1}$
 - d. $y = \frac{x}{x-1}$
 - e. $y = x + 1$
12. A two-digit positive integer whose digits are both not zero is x times the sum of its digits. The integer formed by reversing the digits of the original number is the sum of the digits of the original number multiplied by what?
- a. x
 - b. $11 - x$
 - c. $9 - 2x$
 - d. $1/x$
 - e. None of the above.
13. Two sports teams (team A and team B) are playing a best-of-seven game series. That is, as soon as one team wins four games, the series is over, and they are the champion. The two teams are equally matched and either team will win any given game with probability $1/2$. What is the probability that team A wins the series in exactly five games?
- a. $1/4$
 - b. $1/8$
 - c. $3/32$
 - d. $5/32$
 - e. None of the above.

14. Chad, Carla, Cole, Cliff, and Chloe are each asked to choose a number between 1 and 10. What is the probability that at least two of them will choose the same number?

a. $\frac{1}{2}$

b. $\frac{3}{1000}$

c. $\frac{436}{625}$

d. $\frac{1061}{1250}$

e. none of the above.

15. Three positive integers are in increasing arithmetic progression with a common difference of 3. If the first number is decreased by 2, the second increased by 7, and the third multiplied by 5 and then increased by 4, the new numbers in the given order form a geometric progression. Find the original three positive integers.

a. 5, 8, 11

b. 6, 9, 12

c. 7, 10, 13

d. 8, 11, 14

e. none of the above.

16. The exact value of $\cot(15^\circ)$ is:

a. $\frac{1}{2+\sqrt{3}}$

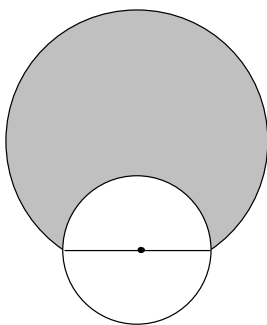
b. $2 + \sqrt{3}$

c. $\frac{1}{3+\sqrt{2}}$

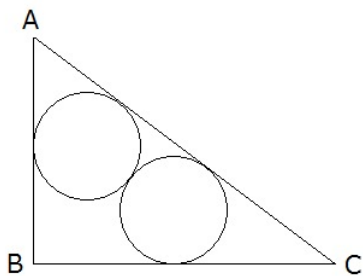
d. $3 + \sqrt{2}$

e. none of the above.

17. Find the area of the shaded region in the following figure. The circles have radius 1 and 2. The smaller circle intersects the larger at 2 points on a diameter of the smaller circle.



- a. $\frac{17\pi}{6} + \sqrt{3}$
- b. $\frac{7\pi}{2}$
- c. $4\pi - \sqrt{3}$
- d. $3\pi + \sqrt{3}$
- e. none of the above.
18. In the following figure, $\triangle ABC$ is a right triangle with $AB = 3$ and $BC = 4$ and $m\angle ABC = 90^\circ$. Two congruent circles, externally tangent to each other, are drawn inside the triangle, with each circle tangent to the hypotenuse and one of the legs. What is the distance between the centers of the two circles?

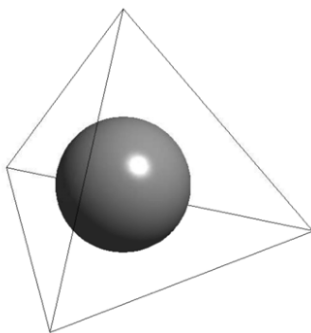


- a. $\frac{1}{3}$
- b. $\frac{5}{7}$
- c. $\frac{10}{7}$
- d. $\frac{11}{9}$
- e. None of the above.

19. If x is positive real number greater than 1 such that $\log_2(\log_8(x)) = \log_8(\log_2(x))$, then what is the value of $(\log_2(x))^2$?

- a. 3
- b. 27
- c. 12
- d. 48
- e. None of the above.

20. A sphere is inscribed in a regular tetrahedron as shown below:



If the length of an altitude of the tetrahedron is 36, what is the length of the radius of the sphere? (Note that the sphere touches each face of the tetrahedron at its centroid. An altitude of the tetrahedron is defined as the distance from any vertex of the tetrahedron to the centroid of the opposite face.)

- a. 6
- b. $12/5$
- c. $24/11$
- d. 9
- e. none of the above.

THE WRITTEN EXAM ENDS HERE.