1. Given the following graph of a function $f$,

![Graph of function $f$]

(4 pt.) a. Find the domain of $f$.

$$\text{dom}(f) = (-\infty, \infty)$$

(4 pt.) b. Find the range of $f$.

$$\text{ran}(f) = [4, \infty) = \{y \mid y \geq -4\}$$

(2 pt.) c. Find $f(6)$.

2. Given $f(x) = x^2$, let $g(x) = f(x - 3) - 4$.

Sketch the graph of $g(x)$ below, labeling the vertex and $x$-intercepts of the graph.

(Graph 8 pt., vertex 2 pt.)

![Graph of function $g(x)$ with vertex and $x$-intercepts labeled]
(10 pt.) 3. Given \( f(x) = x^2 + x \), find the average rate of change of \( f \) from \( x = 2 \) to \( x = 3 \).

\[
\frac{f(3) - f(2)}{3 - 2} = \frac{[3^2 + 3] - [2^2 + 2]}{1} = 12 - 6 = 6
\]

(10 pt.) 4. Given the polynomial function \( P(x) = x^3 - 2x^2 - 5x + 6 \), use the factor theorem to determine whether \( x - 1 \) is a factor of \( P(x) \). If so, express \( P(x) \) as a product of linear factors.

\[
P(1) = 1^3 - 2(1)^2 - 5(1) + 6
\]

\[= 1 - 2 - 5 + 6 = 0\]

Yes, \( x - 1 \) is a factor of \( P(x) \).

Factorization:

\[
\begin{array}{c|cccc}
 & 1 & -2 & -5 & 6 \\
\hline
1 & 1 & -1 & -6 \\
1 & 1 & -6 \\
\end{array}
\]

\[
(x - 1)(x^2 - x - 6) = (x - 1)(x + 2)(x - 3)
\]

5. The Intermediate Value Theorem for Polynomial Functions: If \( f \) is a polynomial function and \([a, b]\) is a closed interval from the domain of \( f \), then \( f \) takes on every value between \( f(a) \) and \( f(b) \) in the interval \([a, b]\).

(6 pt.) a. Use the intermediate value theorem to show that \( f(x) = x^5 - 2 \) has a zero between \( x = 1 \) and \( x = 2 \).

\[
\text{Since } f(1) = 1^5 - 2 = -1 \text{ and } f(2) = 2^5 - 2 = 30
\]

\[\text{and } -1 < 0 < 30, \text{ then there must be a zero between } x = 1 \text{ and } x = 2.\]

(4 pt.) b. Use your calculator to estimate the zero to at least two decimal places of accuracy.

\[
x \approx 1.15
\]
6. Given the rational function \( r(x) = \frac{x+1}{x-2} \)

(2 pt.) a. Find the equation of the vertical asymptote of the graph of \( r(x) \).

\[ x = 2 \]

(2 pt.) b. Find the \( x \) and \( y \) intercept(s) of the graph of \( r(x) \).

\[
\begin{align*}
\text{\( r(0) = \frac{0+1}{0-2} = -\frac{1}{2} \)} \\
\text{\( \frac{x+1}{x-2} = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1 \)} \\
\text{\( \text{y-intercept: } (0, -\frac{1}{2}) \)} \\
\text{\( \text{x-intercept: } (-1, 0) \)}
\end{align*}
\]

(2 pt.) c. Find the equation of the horizontal asymptote of the graph of \( r(x) \).

\[
\begin{align*}
\text{\( r(x) = \frac{x+1}{x-2} = \frac{1 + \frac{1}{x}}{1 - \frac{2}{x}} \Rightarrow 1 \text{ as } x \to \infty \)} \\
\text{horizontal asymptote: } y = 1
\end{align*}
\]

(4 pt.) d. Sketch the graph of \( r(x) \).
7. Given \( f(x) = \cos(x) \) and \( g(x) = x^2 \), find the following.

\[(6 \text{ pt.}) \text{ a. } (f \circ g)(x) = f(g(x)) = \cos(x^2)\]

\[(4 \text{ pt.}) \text{ b. } (g \circ f)(\pi) = g(f(\pi)) = (\cos \pi)^2 = (-1)^2 = 1\]

(10 pt.) 8. Solve the equation. Leave your answer in exact form.

\[ \log(x) - \log(x - 48) = 1 \]

\[ \log \left( \frac{x}{x - 48} \right) = 1 \]

\[ 10 = \frac{x}{x - 48} \]

\[ 10(x - 48) = x \]

\[ 10x - 480 = x \]

\[ -480 = -9x \]

\[ x = \frac{480}{9} \]

Solution set: \( \left\{ \frac{160}{3} \right\} \)

9. The number of bacteria on a certain culture increases exponentially over time. The number \( f(t) \) of bacteria after \( t \) hours is given by

\[ f(t) = 200e^{0.75t} \]

(2 pt.) a. What is the initial number of bacteria? \((t=0)\)

\[ f(0) = 200e^{0.75 \cdot 0} = 200 \]

(4 pt.) b. Estimate the number of bacteria in the culture after two hours. Round your answer to the nearest whole number.

\[ f(2) = 200e^{0.75 \cdot 2} = 896 \]

(4 pt.) c. How long will it take for the number of bacteria to become 4000? Round your answer to the nearest whole number of hours.

\[ 200e^{0.75t} = 4000 \]

\[ e^{0.75t} = \frac{4000}{200} = 20 \]

\[ 0.75t = \ln(20) \]

\[ t = \frac{\ln(20)}{0.75} \approx 4 \text{ hrs} \]
10. Find the inverse function of \( f(x) = \frac{x - 2}{x + 3} \).

\[
  \begin{align*}
    y &= \frac{x - 2}{x + 3} \\
    x &= y - 2 \\
    y &= 3x + 2 \\
    x &= y (1 - x)
  \end{align*}
\]

\[
  f^{-1}(x) = \frac{3x + 2}{1 - x}
\]

11. Solve the right triangle for angle \( \beta \) and sides \( b \) and \( c \), given that \( \alpha = 32^\circ \) and side \( a = 10.2 \). Round sides to one decimal place. (angle 2 pt., sides 4 pt. ea.)

\[
\begin{align*}
  \sin 32^\circ &= \frac{10.2}{c} \\
  \Rightarrow c &= \frac{10.2}{\sin 32^\circ} \\
  \Rightarrow c &\approx 19.2
\end{align*}
\]

\[
\begin{align*}
  \tan 32^\circ &= \frac{10.2}{b} \\
  \Rightarrow b &= \frac{10.2}{\tan 32^\circ} \\
  \Rightarrow b &\approx 16.3
\end{align*}
\]

12. Find the exact values of the following.

(4 pt.) a. \( \sec \left( \frac{5\pi}{3} \right) = \frac{1}{\cos \left( \frac{5\pi}{3} \right)} = \frac{1}{\left( \frac{1}{2} \right)} = 2 \)

(4 pt.) b. \( \cos \left( \frac{7\pi}{6} \right) = \frac{\sqrt{3}}{2} \)

(2 pt.) c. \( \tan(3\pi) = \frac{\sin(3\pi)}{\cos(3\pi)} = 0 \)
13. Given the trigonometric function

\[ f(x) = 2\sin\left(x - \frac{\pi}{2}\right) = 2\sin\left(2x - \pi\right) \]

(2 pt.) a. Determine the amplitude of \( f(x) \).

\[ \boxed{\text{Amplitude} = 2} \]

(2 pt.) b. Determine the period of \( f(x) \).

\[ \frac{2\pi}{2} = \pi \quad \boxed{\text{Period} = \pi} \]

(2 pt.) c. Determine the phase shift of \( f(x) \).

\[ \boxed{\text{Phase shift} = \frac{\pi}{2}} \]

(4 pt.) d. Sketch two periods of the graph of \( f(x) \).
(10 pt.) 14. A ramp is to be built to access a porch that is 2 feet high. How long must the sloped part of the ramp be if it makes an $8^\circ$ angle with the ground?

\[ \sin 8^\circ = \frac{2}{L} \]

\[ L = \frac{2}{\sin 8^\circ} \approx 14.37 \text{ ft} \]

(10 pt.) 15. Verify the following trigonometric identity.

\[ \frac{\cos(t)}{1 - \sin(t)} \frac{\sin(t)}{\cos(t)} = \sec(t) \]

\[ \frac{\cos^2 t - \sin^2 t}{\cos t (1 - \sin^2 t)} = \frac{\cos^2 t - \sin^2 t}{\cos t} = \frac{\cos^2 t}{\cos t} - \frac{\sin^2 t}{\cos t} = \cos t - \sec t \]

(10 pt.) 16. Solve the following equation in the interval $[0, 2\pi)$. State your answers correct to three decimal places.

\[ 7 \cos(x) - 1 = 0 \]

\[ \cos x = \frac{1}{7} \]

\[ x = 1.427, 4.856 \]
17. Find all solutions to the following equation. Give your answers in exact form.

\[ \cot(\theta) = 1 \]

\[ \frac{\cos \theta}{\sin \theta} = 1 \]

\[ \cos \theta = \sin \theta \]

\[ \theta = \frac{\pi}{4} + n\pi, \quad n = 0, \pm 1, \pm 2, \ldots \]

18. Find the exact value of the following expressions.

(a) \[ \cos \left( \tan^{-1} \left( \frac{9}{40} \right) \right) = \frac{40}{41} \]

(b) \[ \sin^{-1} \left( -\frac{\sqrt{2}}{2} \right) = -\frac{\pi}{4} \]

19. If \( \sin(\theta) = -\frac{4}{5} \), and \( \theta \) is an angle in quadrant III, find the exact value of the following.

(a) \( \sin \left( \frac{\theta}{2} \right) \)

\[ \pi < \theta < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \] (quadrant III)

\[ \sin \left( \frac{\theta}{2} \right) = \sqrt{\frac{1}{2} (1 - \cos \theta)} = \sqrt{\frac{1}{2} (1 + \frac{3}{5})} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5} \]

(b) \( \cos(2\theta) \)

\[ \cos(2\theta) = 2\cos^2 \theta - 1 \]

\[ = 2 \left( \frac{3}{5} \right)^2 - 1 \]

\[ = 2 \cdot \frac{9}{25} - 1 = \frac{18}{25} - \frac{25}{25} = \frac{7}{25} \]
20. Refer to the figure below, depicting a triangle with sides shown.

(6 pt.) a. Find the angle (in degrees) opposite the longest side. State your answer to the nearest tenth.

**Law of Cosines**

\[ 14^2 = 7^2 + 11^2 - 2(7)(11) \cos \theta \]

\[ \cos \theta = \frac{26}{-156} \]

\[ \theta = \cos^{-1} \left( \frac{-26}{156} \right) \approx 99.6^\circ \]

(4 pt.) b. Find the area of the triangle. State your answer to the nearest tenth.

\[ A = \frac{1}{2} \times 7 \times 11 \times \sin (99.6^\circ) \]

\[ \approx 38 \]

or using Heron's formula

\[ s = \frac{7 + 11 + 14}{2} = 16 \]

\[ A = \sqrt{16 \times (16-7)(16-11)(16-14)} \]

\[ \approx 37.94 \approx \boxed{38} \]